Math 10B with Professor Stankova
Quiz 8; Tuesday, 3/19/2019
Section #203; Time: 11 AM
GSI name: Roy Zhao **Name:** ____________________

Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. **TRUE** False The Law of Large Numbers tells us that for fixed $\epsilon > 0$, the probability $P(|\bar{X} - \bar{\mu}| < \epsilon)$ goes to 1 for large n.

   Solution: The version stated in class says that the stuff outside of ϵ goes to 0 which means the stuff inside ϵ must go to 1 minus that or 1.

2. True **FALSE** For large n, the average random variable $\bar{X}$ is nomrally distributed.

   Solution: The CLT tells us that $\bar{X}$ is **approximately** normally distributed.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) Suppose that a random basketball fan has a 10% chance of liking the Lakers and this probability is independent of any other fan.

   (a) (2 points) Choose a random fan. Let X be the random variable that outputs 1 if they like the Lakers and 0 otherwise. What is $E[X]$ and $SE(X)$? (Simplify your answer)

   Solution: This is a Bernoulli trial with probability of success $p = 0.1$. Then $E[X] = p = 0.1$ and $SE(X) = \sqrt{Var(X)} = \sqrt{p(1-p)} = \sqrt{0.1(0.9)} = 0.3$.

   (b) (4 points) What is the probability that in a party of 25 fans, at most $4\%(=\frac{1}{25})$ of them like the Lakers? (You do not need to simplify your answer)

   Solution: This is repeating 25 Bernoulli trials so this is a binomial distribution. We have $n = 25, p = 0.1$ and the probability that at most 1 likes the Lakers is

$$f(0) + f(1) = \binom{25}{0}(0.1)^0(0.9)^{25} + \binom{25}{1}(0.1)^1(0.9)^{24}.$$

(c) (4 points) Use the CLT to approximate the probability that at most 4% of the 25 fans like the Lakers. (Hint: $z(1) = 0.3413$)

Solution: Let $\bar{X}$ be the average of the X_i for the fans, which is the percentage of fans who like the Lakers out of a party of $n = 25$ of them. Then $\bar{\mu} = \mu = 0.1$ and $\bar{\sigma} = \sigma/\sqrt{n} = 0.3/\sqrt{25} = 0.06$. We want to calculate $P(\bar{X} \leq 0.04)$. The CLT tells us that $\bar{X}$ is approximately normally distributed so using z scores, this probability is approximately $1/2 - z(|0.04 - 0.1|/0.06) = 1/2 - z(1) = 0.1387$.